Liminal Tones: Swarm Aesthetics and Materiality in Sound Art

Mahsoo Salimi[1] and Philippe Pasquier[2]

Simon Fraser University, Surrey, BC V3T0A3, Canada

[1] PhD Candidate, Metacreation Lab for Creative AI
[2] Associate Professor, Director, Metacreation Lab for Creative AI
`salimi@sfu.ca, pasquier@sfu.ca`

Abstract: The application of swarm aesthetic in music composition is not new. Artistic swarm application has resulted in complex soundscapes and musical compositions. However, sound composition using physical swarm agents has not been extensively studied. Using an experimental approach, we create a series of sound textures know as Liminal Tones (B/ Rain Dream) based on swarming behaviours. We study the influence of different materials and emergent patterns and evaluate the acoustic properties of different materials such as wood, ceramic or granite, and effect of imperfections of the physical agents on the overall aesthetic quality. Finally, we consider the historical and theoretical foundation of swarm music, the role of materiality and actions in sound, and challenge the traditional perception of sound as an immaterial art form.

1   Introduction

Swarm systems inspired by swarm intelligence and natural ecosystems (e.g., social insects) are a unique frontier for art. Many artists utilize swarm principles such as indirect communication, self-organization and emergent behaviours to create musical compositions, soundscapes and sonic environments. SWARMUSIC [3] is a system that uses swarm behaviour to create music. It is an interactive music improvisation tool with multiple swarms of particles as musical events that move in a virtual 3D space by utilizing Boids flocking algorithm [24]. Bisig et al. [2] created a series of experimental projects known as Interactive Swarm Orchestra (ISO) and Interactive Swarm Space (ISS). The ISO system explores flocking algorithms to control sound synthesis and sound spatialization. The ISS is a MIDI-based virtual orchestra involving meaningful interactions between artificial swarms and composers to generate artistic expression. Bisig et al. also have explored multi-modal feedback and audio-visual spatialization or creative engagement using swarm techniques.

Expanding on previous work, Davis and Karamanlis [9] added a controllable leader to typical Boids simulations for musical swarms. The leader agent lets the user directly control the behaviour of the other agents and the overall movement of the flock. In a different approach, Jones [18] introduced AtomSwarm. This is a framework for sound-based performance that uses swarm dynamics with genetically-encoded behaviours, artificial pheromones and imitations. The result is a complex sonic ecosystem capable of sonic spatialization and self-organizing regulation. Flock to Music [6] is a real-time improvisation tool that simulates the behaviour of the Boids as compositions with musical parameters.

Despite the broad interest in swarm music, most experiments to date utilize artificial swarms and software simulations. To our knowledge, there has been little exploration of physical swarming agents. Blackwell [4] provides a comprehensive review of swarm music.

Self-organization is a unique and complex collective behaviour common in swarms. It results from simple and local interactions between agents (members of the group) and emerges at the colony level. Self-organization of social insects usually happens via stigmergy, an indirect communication strategy through the environment [16]. Stigmergy results in a complex emerging intelligence at the colony level without the need for planning, control or direct communication between agents. However, with no prior knowledge about the sources, systematic searches become less effective and social insects often use other searching mechanisms known as random walk. There are several random walk variants, including Stigmergic Random Walk (SRW), Correlated Random Walk (CRW) and Lévy Walk. Random Walks are commonly used for artistic experiments and in swarm robotics particularly if the robots have limited individual abilities (e.g., local sensing, memory or processing power). Considering the limitation of our BBots, we have leveraged stigmergic foraging behaviours and variations of random walks to create sound compositions.

Non-human Sounds – Sound as Action. Using mechanical devices and computer-controlled sound objects is not new in sound art. However, there is a new series of work involving mechanical and glitch sounds. Such works have focused on exploring repetitive sonic processes and events with mechatronic mediation. Mechanical/rhythmic actions, sound experiences and the ontological properties of non-human sounds are more important than traditional interventions. The investigation of space as a compositional element, modulated by movement, offers new idiosyncrasies and aesthetic potentials for musical creation.

Over the past two decades, robotic and mechatronic interventions have become prominent aesthetic elements in the work of composers and sound artists. These implications include electro-acoustic experimentation, sonic environments, sound sculptures and the use of drumming apparatuses. However, mechatronic systems used for musical creations can have many different aesthetic roles. Some artists use motion, direction and distance of sound as compositional means and sound spatialization. Composer and sound artist Trimpin employs the visual, spatial and kinetic properties of sound in his works (e.g., Conloninpurple, 1997; Sheng High, 2004). Other artists use

them to evoke memories and imaginary environments and to stimulate different emotions [11].

Sound artists Peter Bosch and Simone Simons [5] explore the spatial characteristics of sound in their kinetic sound project Cantan un Huevo (2000). They use glass bottles, containers and metal springs as sound objects. The distribution of the sound sources in the space is an integral part of their work and results in different acoustic experiences in different parts of the space.

Other artists use similar sound objects distributed evenly across space in their work. Pe Lang and Zimoun [22, 32] create sound sculptures and installations with rhythms and flow, using basic mechanical components (as sound objects) in large numbers. In their practice, together and individually, they create analog rhythms, textures and flow to study the creation and degeneration of sonic spaces. Inspired by generative systems and swarm behaviours, their works display simplicity and complexity. The emergent and intricate behaviours of these sound objects (in sound and motion) appear to be organic or alive and sound like "the acoustic hum of natural phenomena" [27].

Building on our previous work [25] on swarming techniques and robotic interventions in sound art, and inspired by Pe Lang and Zimoun's artworks [22, 32] and Blackwell's SWARMUSIC [4], we introduce Liminal Tones (B/ Rain Dream) as an experimental sound art project/tool. Our goal is to demonstrate the importance of actions, materials and acoustic media in sound texture, using multi-bodies (a swarm of physical agents) and challenging the traditional perception of music as an immaterial art form.

Previously we used digital mediation and PSO-PID controller to derive the movement of DC motors and generate sound, but here we use an analogue approach and swarming BBots to generate sound textures and further investigate the influence of materiality and robotic intervention to generate novel sound textures (acoustic aesthetics). So, we present Liminal Tones (B/ Rain Dream) and show the results and analyze the influence of different materials on the aesthetic quality of sound textures in Section 2. Then, we follow up with a discussion of the relationship between order, chaos and emergent behaviours of Liminal Tones (B/ Rain Dream) in Section 3. Finally, we explain the underlying concepts of swarm aesthetics for musical creation and discuss our future plans in Section 4.

2 Methods

2.1 Concept

Liminal Tones (B/ Rain Dream) is a series of sound textures made by a group (5–10) of BBots (as sound objects) that move, twist and turn on the ground to generate sounds (BBot is a modified version of vibration-driven Bristlebot [15, 1] with no brush). Inspired by Pe Lang and Zimoun's sound sculptures [22, 32], we used DC vibrator motors, wires and electrical circuits to create the BBots and control their motion and sound. Liminal Tones (B/ Rain Dream) demonstrate collective behaviours while embracing randomness and imperfections (due to battery degradation and DC perturbation). The resulting sound textures are both organized and chaotic. Liminal

Tones (B/ Rain Dream) can be viewed as an experimental tool for emergent behaviours and materiality in sound art [13] rather than an artwork. Using different materials (as surface) and tuning the initial conditions (placement, speed, direction), we were able to create different sound textures despite the identical shape and properties of BBots. Listening to the textures, one can recognize rhythms such as the clicking of a drum or natural sounds (e.g., raindrops on the metal roof). Audio samples can be found on our website [21].

2.2 Model

BBots (sound objects) exhibit complex movements similar to the stigmergic foraging behaviour of ants, in two phases. First, sound objects demonstrate Lévy Walk with high power and speed. Over time, the sound objects cycle to Brownian Motion as the battery degrades (with lower speed).

Phase 1 – Lévy Walk. At the start each BBot move quickly with large step-size similar to Lévy Walk motion – a modification of the standard random walk in which the step size has a heavy-tailed distribution [31]:

$$P(s) = s^{-\mu} . \tag{1}$$

where s is the step size with $1 < \mu \leq 3$. With increasing values of μ the movement becomes less super-diffusive (due to jumps with heavy-tail distribution) and more Brownian. Individual objects with super-diffusive movement paths will appear to move faster than those with normally diffusive (Brownian) or sub-diffusive movements [31]. Therefore, Lévy walks represent a spectrum of random walks, with ballistic motion at one extreme ($\mu > 1$) and Brownian Motion ($\mu \simeq 1$) at the other.

Formal Asymptotics. We used 5 BBots as sound objects with DC perturbation ranging between 1.5–3V. BBots move with a random heading and a step length selected from a power-law distribution with parameter μ. The periodic vibration of DC motors paired with a friction mechanism lead to a propulsion interaction between the sound objects and the environment, alternating between high friction in some parts and low friction in others. BBots have a body with a rotational spring of stiffness k and are in frictional contact with the surface without any legs.

The force (f_Ω) resulted from the body mass oscillation and frequency Ω drives the internal movement of the sound objects. The modulation of friction of BBots results from the oscillations of the normal forces and leads to a stick-slip motion. DeSimone and Tatone [10] modelled the tangential frictional force by:

$$F = -\mu N \dot{x} . \tag{2}$$

where N is the normal reaction force, $\dot{x}$ is the velocity (denoted with a dot with respect to time), and μ is a constant. For simplicity, we assume that rotations of the BBots are not allowed and they are always in contact with the ground with two degree

of freedoms: horizontal movement and deviation ϕ from the rest angle $\alpha = 0$. Therefore, the motion equation is as follows [8]:

$$M\ddot{x} = -\mu N(t)\dot{x} . \qquad (3)$$

$$M\ddot{y} = N(t) - M_g + f_\Omega(t) . \qquad (4)$$

$$k_\varphi = N(t) L \sin(\alpha + \varphi) - \mu N(t) \dot{x} L \cos(\alpha + \varphi) . \qquad (5)$$

where N is the normal reaction force ($N = \sum_{i=1}^{m} N_i$), and M_g is the body mass. We consider the following ansatz for the normal force:

$$N(t) = N^* + \tilde{N} \sin\Omega t . \qquad (6)$$

$$N^* = M_g . \qquad (7)$$

$$\tilde{N}/N^\wedge * = \eta \ll 1 . \qquad (8)$$

where η is the ratio between the amplitude of a harmonic ($\tilde{N}$) and the average normal force (N^*) and usually smaller than 1. To normalize the dynamic variables, we consider the following constants:

$$\sigma = \sin(\alpha) . \qquad (9)$$

$$\chi = \cos(\alpha) . \qquad (10)$$

$$f_\Omega(t) = N^* f(\Omega t) . \qquad (11)$$

$$\Omega = \sqrt{(k/M)} \ \omega/L\chi . \qquad (12)$$

where f and ω are the normalized force and frequency. Applying all the definitions above we can rewrite equations (3) and (5) as the equivalent system in respect to dynamical variables (θ, w).

$$\theta = n\tau \frac{\sin(\alpha+\theta)}{\sigma} - \xi n\tau \left(w + \dot{\theta} \frac{\cos(\dot{\alpha}+\theta)}{\chi}\right) \cos(\alpha + \theta)/\chi . \qquad (13)$$

$$\dot{w} = -\lambda n\tau(w + (\dot{\theta} \ (\cos(\alpha + \theta))/\chi)^\cdot) . \qquad (14)$$

where $\tau = \Omega t$, $\xi = \frac{\mu N^* L^2 \cos \alpha \Omega}{k}$ and $\lambda = \frac{\mu N^*}{M\Omega}$.

Phase 2 – Brownian Motion. After a few minutes, BBots move slowly with smaller step sizes as the batteries degrade. In this phase each BBot acts as a particle with a

normalized step-size distribution similar to Brownian motion and constantly moves in random directions.

The Brownian motion is a complex random process with noise. There are different methods to formulate the Brownian motion in terms of the evolution of a nonstationary probability and here we use Langevin and Fokker-Plank equation [19, 23] to study the evolution of the velocity distribution and interactions between the environment and Brownian agents. The dynamics and speed fluctuation of the Brownian particle are defined as:

$$\dot{x} = v . \tag{15}$$

$$\dot{v} = -\gamma(x, v)v + F(t) + \xi(t) . \tag{16}$$

where $F(t)$ represents a random external force, m and v the mass and the velocity of the particle, $\xi(t)$ is a Gaussian noise, α is the friction constant and $\gamma = \frac{\alpha}{m}$.

For simplicity, we assume there is no external forces, and therefore $F(t) = 0$. The Brownian particle with the state space (x, v) has a distribution probability $\rho(x, v, t)$ as follows [23]:

$$\partial/\partial t \, \rho(x, v, t) = -\nabla(\rho(x, v, t)(\dot{x}, \dot{v})) . \tag{17}$$

$$\partial/\partial t \, \rho(x, v, t) = -\partial/\partial x \, (\rho\dot{x}) - \partial/\partial v(\rho\dot{v}) . \tag{18}$$

To simplify the equation, operators A and B are defined as:

$$A = v \, \partial/\partial x - \partial/\partial v \, (\gamma(x, v)v) - \gamma(x, v)v \, \partial/\partial v . \tag{19}$$

$$B = \xi(t)\frac{\partial}{\partial v} . \tag{20}$$

Hence:

$$\partial/\partial t \, \rho(x, v, t) = -A\rho - B\rho . \tag{21}$$

3   Results

In this section we present the initial results of Liminal Tones (B/ Rain Dream) and step-length distributions for each phase of the model scheme. We analyze samples taken from different intervals and compare the sound quality of different motion (Lévy Walk or Brownian) and the surface material in Fig. 1 and Fig. 2. First, we show examples of movement trajectories of BBots of different surfaces (wood, ceramic, granite) and the dependence of those trajectories on control parameters $\varepsilon \to \hat{f}(x, y)$ and DC motor speeds. When $\mu > 1$ and BBots have high turning angle and speed (interacting with the environment), the motion is ballistic with long, straight movements and many short

steps as shown in Fig. 1. In contrast, when $\mu \simeq 1$ the motion is Brownian as shown in Fig. 2.

The movement trajectories (different μ) depends on the distribution of step lengths. With smaller and fixed μ, the step-length distribution is more stable (Cauchy distribution). With random or higher μ values, the step-length distribution becomes Gaussian. Moreover, the motions result from turning angles $\Delta\theta_t$ over time (t). When the value of $\Delta\theta_t$ is close to zero for a long time, BBots move in a straight line. In contrast, when $\Delta\theta_t$ fluctuates dynamically, BBots twist and turn many times.

To evaluate the quality of the generated sound textures, we compare them to natural ambient sounds with similar audio profiles. Usually, BBots generate rhythmic patterns with high jumps between different frequencies. This would be similar to the rhythmic pattern of heavy hail and the noisy profile and calming pattern of sleet, as illustrated in Fig 3. To qualitatively assess the role of materiality in sound, we compare the spectrum of acoustic sound objects in relation to different materials, and their pitch and timbral aesthetic for 12 sound textures [21] as shown in Fig 4. Here, vertical lines represent the rhythmic structures and horizontal lines represent the harmonic structures. For some sound categories, the audio samples are noisy, meaning most frequencies are present. Other categories have fewer frequencies and show step intervals and rhythmic cycles which resulted from vibrating patterns, turn and twist of motors, or errors (on-off interruptions). The speeding patterns can also be identified where the sound amplitudes vary due to power fluctuations of the batteries. Notably, each material shows different music signatures. For example, wood resonates at higher frequencies while ceramics absorb sounds and do not resonate as much (low, mid frequencies).
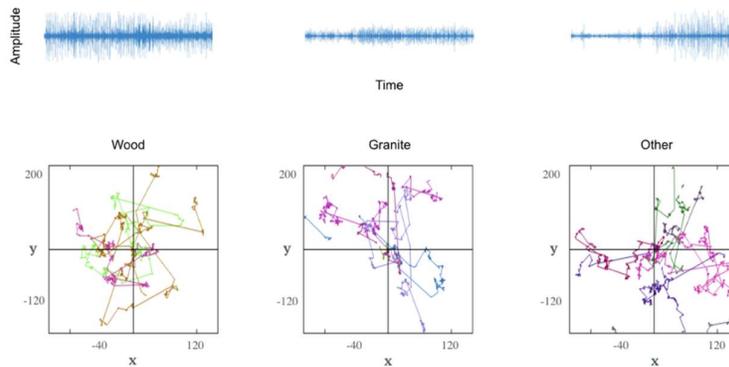

Fig. 1: Model scheme and examples of trajectories for 5 BBots and 10,000 steps with fixed step distribution and high-speed during Phase 1 which follows a ballistic Lévy Walk. Different colours correspond to each BBot and its initial conditions (placement, speed, direction). When the value of $\Delta\theta_t$ is close to zero for a long time, the BBots move in a straight line with short steps in between.
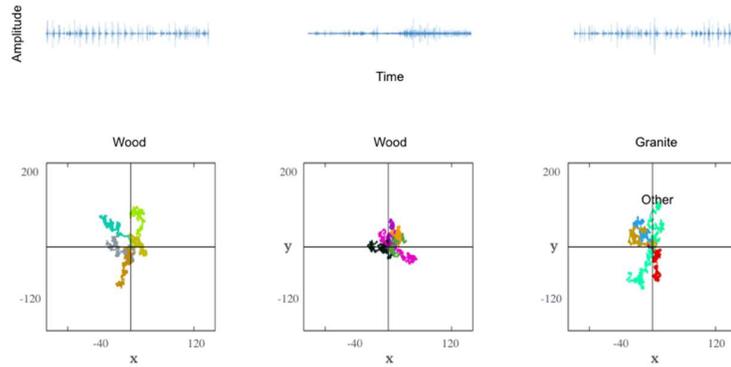
Fig. 2: Model scheme and examples of trajectories for 5 BBots and 10,000 steps. There is random step distribution and low-speed movement during Phase 2, similar to Brownian Motion. The internal dynamics x and y produce agent movements in 2D space. Movement is produced by turning angles $\Delta\theta_t$ over time (t). The trajectory of each BBot in a 2D space is represented by different colours corresponding to each BBot and its initial conditions (placement, speed, direction).
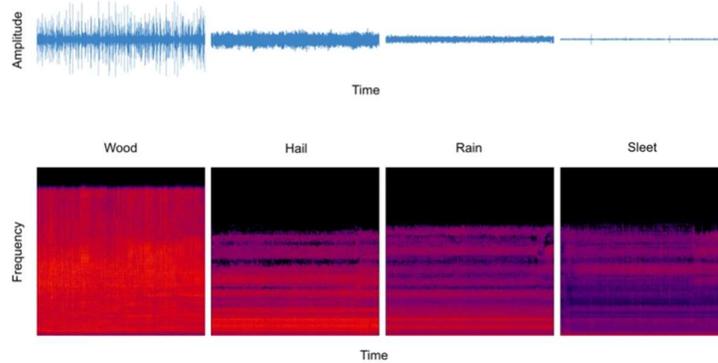

Fig 3: Comparison of audio samples (left) with natural sounds (hail, rain, sleet). Examples were selected to be roughly similar in sound textures. The top row shows the waveforms. Note that our sample is more extremely periodic with high jumps compared to the other three. The bottom row shows the spectrograms. Here, the vertical lines represent step intervals. Note the constant tones around mid-levels in rain and the noisy profile of sleet sound.
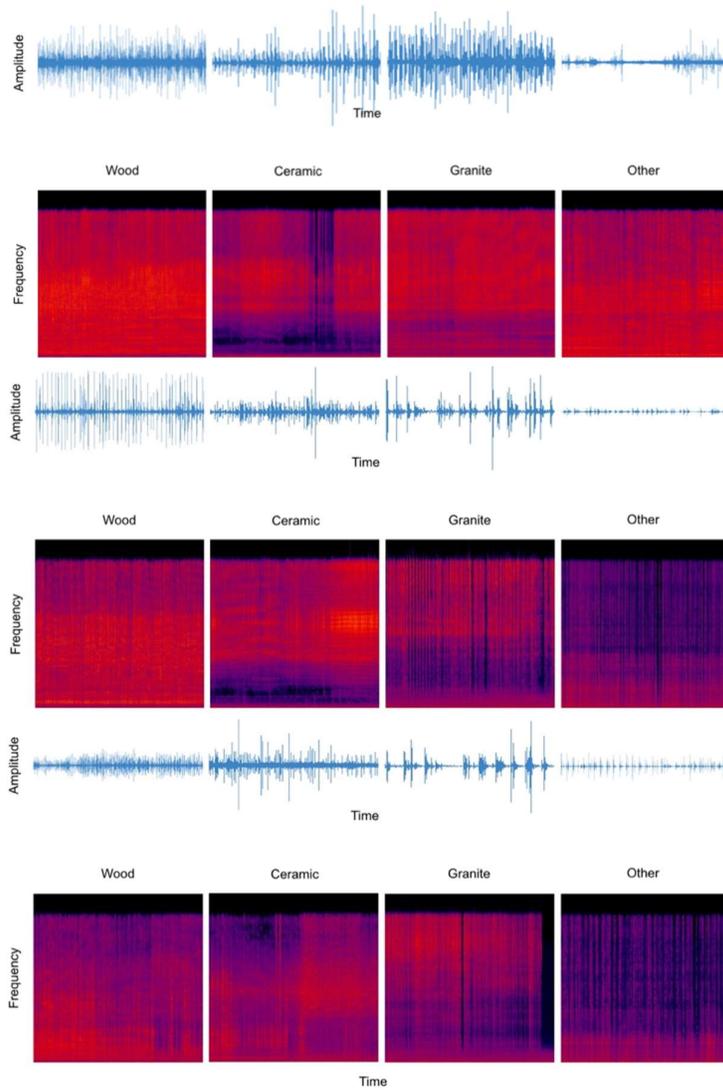
Fig 4: Spectrograms of 12 sound samples (each ranging from 15-30 seconds). Note the constant noisy profile of wood and the mid-level frequencies and orders of ceramics, and resonance of granite. Some samples have different characteristics such as rhythmic patterns and high-low passes. Others are noisy with a wide range of pitch and timbral qualities, which creates unique sound textures [21].

4   Discussion and Future Works

Swarm intelligence is one of the most beautiful and unusual phenomena in nature. It is the product of the interaction between a group of decentralized agents and their environment. Widely recognized examples of swarms include but are not limited to bird flocking, bacterial growth, fish schooling, and the societal superorganisms of ant colonies (i.e., foraging). Due to their aesthetic qualities, swarm systems inspired by swarm intelligence and natural ecosystems present unique frontiers for art domains such as visual art [28, 17] and sound composition [2, 3, 6].

Swarm aesthetics are mostly concerned with form, the collective patterns of artificial swarm agents, and intuitive visual and sonic representations in digital forms. There is a gap in the research and practice of using swarm techniques to create sounds mediated by robotic actions and spatio-temporal processes resulting from: multiple interactions, amplification of fluctuations, or randomness between physical agents (sound objects). We propose Liminal Tones (B/ Rain Dream) as a tool to create sounds from actions (of multiple sound objects) and explore swarm aesthetics in sound.

4.1   Order and Chaos – Sound as Emergence

Chaos theory and the study of complex systems (nonlinear dynamics), provide a framework for thinking about constant tensions and emergence from chaos and order. Deterministic and dynamic systems regardless of their subject matter have universal characteristics, including repetition, self-organization, emergence, feedback loop and unpredictability. Chaos theory focuses on simple systems with unpredictable and emergent behaviours. Complexity theory focuses on complex systems that have numerous interacting parts which are often self-organized and unexpected. In such systems, emergent patterns arise from simple rules, local interactions between the individual elements (or agents) and adaptive behaviours.

Not surprisingly, many artists use multi-agent systems and emergence in music improvisation, compositions and sound art. Despite the emergent behaviours of dynamic systems, artists can control the musical outputs subject to the complexity of the rule set and important variables. Manual control of interconnected systems such as music generative systems is almost impossible because each agents' every movement is affected by other agents. For more control, artists use simple computational models such as Cellular Automaton [14, 5], swarming techniques [3, 17, 18, 28, 30] or abstract constraints [2, 4, 20, , 22, 32].

Throughout the past decades of sound art, there have been a few artists who applied emergence and chaos principles in their work without any digital mediation. Joe Jones, and more recently Zimoun and Pe Lang [29], use simple elements such as motors, wires and solenoids to create sound sculptures and installations. The rhythm and flow in these sonic environments result from repetition, randomness and imperfections or glitches. Zimoun and Pe Lang, together and individually, study the creation and degeneration of patterns. Inspired by generative systems and swarm behaviours, their works display both simplicity and complexity. Here complexity grows from simple rules with some randomness and emphasizes their oppositional position of order and chaos [26].

Inspired by current artistic applications and the rich aesthetic qualities of swarms, we explore robotic interventions and the role of materiality in sound art to create novel sound textures with different pitch and timbral qualities.

4.2   Future Works

While While experimenting with different setups for Liminal Tones (B/ Rain Dream), we tested the use of physical swarming bodies to create sound. To achieve different aesthetic qualities, we explored chaotic and random behaviours, and embraced imperfections and error (due to battery degradation and DC perturbation). Liminal Tones (B/ Rain Dream) that resulted are a critical reflection of a still-emergent field of work.

With respect to our future work, our plan is to investigate multi- swarms (with different sound qualities) and large numbers of BBots (50 or more) to explore collective behaviours, and swarm aesthetics with wide timbral and frequency range, and mechanical tones.

References

1. Becker, Felix, et al. "On the mechanics of bristle-bots-modeling, simulation and experiments." *ISR/Robotik 2014; 41st international symposium on robotics*. VDE, 2014.
2. Bisig, Daniel, Jan C. Schacher, and Martin Neukom. "Flowspace-A Hybrid Ecosystem." In NIME, pp. 260-263. 2011.
3. Blackwell, Tim M., and Peter Bentley. "Improvised music with swarms." In Proceedings of the 2002 Congress on Evolutionary Computation. CEC'02 (Cat. No. 02TH8600), vol. 2, pp. 1462-1467. IEEE, 2002.
4. Blackwell, Tim. "Swarming and music." In Evolutionary computer music, pp. 194-217. Springer, London, 2007.
5. Bosch, Peter, and Simone Simons. "Our music machines." Organised Sound 10, no. 2 (2005): 103.
6. Bouchard Homepage, https://www.livestructures.com/flock-to-music/, last accessed 2021/04/20.
7. Candy, Linda, Ernest Edmonds, and Fabrizio Poltronieri. *Explorations in art and technology*. London: Springer, 2002.
8. Cicconofri, Giancarlo, and Antonio DeSimone. "Motility of a model bristle-bot: A theoretical analysis." *International Journal of Non-Linear Mechanics* 76 (2015): 233-239.
9. Davis, Thomas, and Orestis Karamanlis. "Gestural control of sonic swarms: Composing with grouped sound objects." (2007).
10. DeSimone, Antonio, and A. Tatone. "Crawling motility through the analysis of model locomotors: two case studies." The European Physical Journal E 35, no. 9 (2012): 1-8.
11. Di Bona, Elvira. "Towards a rich view of auditory experience." Philosophical Studies 174, no. 11 (2017): 2629-2643.
12. Dorin, Alan. "Aesthetic fitness and artificial evolution for the selection of imagery from the mythical infinite library." In European Conference on Artificial Life, pp. 659-668. Springer, Berlin, Heidelberg, 2001.

13. Flø, Asbjørn Blokkum. "Materiality in Sound Art." Organised Sound 23, no. 3 (2018): 225.
14. Gage, Dustin, Elizabeth Laub, and Briana McGarry. "Cellular automata: is rule 30 random." In Proceedings of the Midwest NKS Conference, Indiana University. 2005.
15. Giomi, L., N. Hawley-Weld, and L. Mahadevan. "Swarming, swirling and stasis in sequestered bristle-bots." Proceedings of the Royal Society A: Mathematical, Physical and Engineering Sciences 469.2151 (2013): 20120637.
16. Grasse, Pierre-P. "Reconstruction of the nest and coordination between individuals in terms. Bellicositermes Natalensis and Cubitermes sp. the theory of stigmergy: test interpretation of termite constructions." Soc. Insect 6 (1959): 41-80.
17. Jacob, Christian J., Gerald Hushlak, Jeffrey E. Boyd, Paul Nuytten, Maxwell Sayles, and Marcin Pilat. "Swarmart: Interactive art from swarm intelligence." Leonardo 40, no. 3 (2007): 248-254.
18. Jones, Daniel. "AtomSwarm: a framework for swarm improvisation." In Workshops on Applications of Evolutionary Computation, pp. 423-432. Springer, Berlin, Heidelberg, 2008.
19. Klimontovich, Yu L. "Nonlinear brownian motion." Physics-Uspekhi 37, no. 8 (1994): 737.
20. McCormack, Jon. "Evolving sonic ecosystems." Kybernetes (2003).
21. Metacreation Homepage, https://metacreation.net, last accessed 2021/04/20.
22. Pe Lang Homepage, https://www.pelang.ch/pelang.html, last accessed 2021/04/20.
23. Radpay, Parham. "Langevin Equation and Fokker-Planck Equation." (2020).
24. Reynolds, Craig W. "Flocks, herds and schools: A distributed behavioral model." In Proceedings of the 14th annual conference on Computer graphics and interactive techniques, pp. 25-34. 1987.
25. Salimi, Mahsoo., and Philippe Pasquier. "Exploiting Swarm Aesthetics in Sound Art", Art Machines 2: International Symposium on Machine Learning and Art 2021, Art Machines 2 (2021).
26. Satin, Seema, and A. D. Gangal. "Random walk and broad distributions on fractal curves." Chaos, Solitons & Fractals 127 (2019): 17-23.
27. Schlatter, N. Elizabeth, Richard Waller, and Sarah Matheson. "Flow, Just Flow: Variations on a Theme." (2013).
28. Shiffman, Daniel. "Swarm." In ACM SIGGRAPH 2004 Emerging technologies, p. 26. 2004.
29. Stoddart, Madeleine M. "Swiss/Mecha-Swiss: An Investigation Into the Kinetic, Sonic and Entropic Oeuvre of Zimoun." PhD diss., University of London (Courtauld Institute of Art), 2015.
30. Urbano, Paulo. "Playing in the pheromone playground: Experiences in swarm painting." In Workshops on Applications of Evolutionary Computation, pp. 527-532. Springer, Berlin, Heidelberg, 2005.
31. Viswanathan, Gandimohan M., Sergey V. Buldyrev, Shlomo Havlin, M. G. E. Da Luz, E. P. Raposo, and H. Eugene Stanley. "Optimizing the success of random searches." Nature 401, no. 6756 (1999): 911-914.
32. Zimoun Homepage, https://www.zimoun.net, last accessed 2021/04/20.